# Enhance seismic interferometry signal using an adaptive FK filter

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#### Summary

An adaptive FK filter is implemented to enhance reflectivity extracted from ambient noise data. As proved by the application to synthetic noise data, this technique can significantly reduce the time length needed to extract reflectors.

# Introduction

In recent years, seismic interferometry (SI) has been of growing interest in both the earthquake seismology and the reflection seismology. In earthquake seismology, SI has been successfully applied to extract surface waves and to invert for earth structure in a continental scale [*Campillo & Paul*, 2003, *Moschetti et al* 2007 and *Liang & Langston* 2008]. The major focus of SI in the reflection seismology field has been to extract reflectivity from ambient noise [e.g., *Claerbout* 1968, *Draganov et al*, 2006, *Artman* 2006]. The concept of SI can also be applied to interpolate, extrapolate and redatum seismic data [*Shuster* 2009].

The rigorous SI theory presented by *Wapenaar* [2004], *Snieder* [2004] and *Shuster* [2009] established the equivalence between the cross-correlation function and the Green's function between two receivers. In this paper, instead, we use the more straightforward relationship between the transfer function and the cross-correlation function to introduce a new method using FK filter to enhance SI signals. Synthetic study shows that this new method significantly reduces the time length needed to extract reflected signals.

### Theory

As illustrated in the Figure 1, the major purpose of the SI in the reflection seismology is to extract the waves propagating between receiver A and B along the blue dashed path. Based on the Snell's law, waves of our interest travel with a certain apparent velocity (c) defined by the offset of the two receivers and the property of the media. A real source S emits energy in all directions, including direct waves (gray lines) to two receivers and other scattered waves (the red path as an example). But only the wavelet traveling along the red path (S-R-A) will sample both receivers and reflect off the target reflector once. In another word, only the part of the wavefield traveling with an apparent velocity c will contribute to the SI signal. We define the wavelet arriving at receiver A along this path as a virtual source (the green N-shape wavelet).  $h_A^S(\mathcal{C})$  be a virtual source at receiver A, its response (blue

M-shape wavelets) on receiver B is then  $T_{BA}(c)h_A^S(c)$ with  $T_{BA}(c)$  the transfer function representing the reflection from A to B along the dashed blue path. The variable "c" represents the path-dependence of these terms. All these terms are defined in the frequency domain and the frequency variable is omitted for simplicity. If we define the summation of all such virtual sources due to real sources distributed in whole space and time as the accumulated source (AS) at receiver A (the green trace) and denote it as  $H_A^S(c) = \sum h_A^S$ , its response on B (blue trace) can then be represented as  $T_{BA}(c)H_A^S(c)$ . The accumulated source at receiver B,  $H_B^S(c)$  (red traces with V-shape wavelet), and its response  $T_{AB}(c)H_B^S(c)$ (magenta trace with W-shape wavelets) can be defined similarly. Based on the above definition, the full wavefields at receiver A ( $\phi_A$ ) and B ( $\phi_B$ ) can be expressed as equation (1) and (2), respectively,



Figure 1: Wave Field Division (upper panel) and the geometry of receivers and model. Wave traces on the topleft and top-right are divided three parts of the wavefield at receiver A and B, respectively. Two receivers are placed on surface (Black Line). A reflector (Solid blue line) is located below the surface. S and R denote a source and a scatter, respectively.

$$\phi_{A} = H_{A}^{S}(c) + T_{AB}(c)H_{B}^{S}(c) + N_{A}$$
(1)  
$$\phi_{B} = T_{BA}(c)H_{A}^{S}(c) + H_{B}^{S}(c) + N_{B}$$
(2)

with  $N_A$  and  $N_B$  the background noise and other wavefield not related to the target reflection at receiver A and B, respectively. Obviously, the wavefield division (WFD) presented in equation (1) and (2) is subject to the geometry of the target reflector and the relative locations of two receivers. The cross-correlation of the two wave-fields at A and B can now be found as equation (3):

$$C_{AB} = T_{AB} \left( H_B^S H_B^{S*} \right) + T_{BA}^* \left( H_A^S H_A^{S*} \right) + C_{AB}^{bg}$$
(3)

with "\*" representing conjugate and  $C_{AB}^{bg}$  representing the cross-correlation of the background noise and other terms. For clarity, we have dropped the apparent velocity "c" from all terms. Similarly, the auto-correlation of the wavefield at receiver A can be represented as the equation (4).

$$C_{AA} = T_{AA} \left( H_A^S H_A^{S*} \right) + C_{AA}^{bg}$$

$$\tag{4}$$

Equation (3) suggests that only when the accumulated source function  $H_A^S(c)$  or  $H_B^S(c)$  is white, the equivalence between the cross-correlation and the transfer function can be established. If the real sources are dominantly from the right side of the receiver B, the transfer function  $T_{{\scriptscriptstyle A}{\scriptscriptstyle B}}$  will be weighted more than  $T_{{\scriptscriptstyle B}{\scriptscriptstyle A}}\,$  , i.e,  $\left\|H_{B}^{S}H_{B}^{S*}\right\| > \left\|H_{A}^{S}H_{A}^{S*}\right\|$ . As a result, the signal on the crosscorrelation function will be stronger on one side than another. Such asymmetric cross-correlation functions are very well illustrated by the surface waves extracted from ambient noise by Liang & Langston [2008].

Equation (3) and (4) also suggest that, to enhance the transfer function is equivalent to enhancing the accumulated source at either receiver. That is, only when

We use synthetic examples in the next section to illustrate these ideas.

#### Synthetic Examples

The setup of synthetic experiments is shown in the Figure 2. We generate multiple time segments (15 minutes each) of noise data. For each time segment, waves generated by 1000 explosive sources are added to a random background wavefield with amplitudes ranging between -1 and 1. The peak amplitude of synthetic waveforms for each source ranges between 0.5 to 2.0. All sources are distributed randomly both in time and in space (along the source line in Figure 2). A reflector is located at 2km depth and the two way zero-offset travel time is 2 seconds.

As discussed before, only vertically traveling waves rebound up and down between the receiver and the reflector. Thus, to enhance the  $(H_A^S H_A^{S*})$  in the equation (4), a narrow band FK filter is applied to suppress waves with low apparent velocity. The summation of autocorrelation fuctions of multiple time segments are plotted in the Figure 3. Without using FK filter, it takes at least 250 hours (Figure 3b) of data to bring up clear signals of the target reflector. On the other hand, with the FK filter (Figure 3c) applied before auto-correlation, 25 hours of data yields even stronger SI singals than that from 250 hours of orginal data (Figure 3b).



layers are given as Vp1, Vs1, Vp2 and Vs2, respectively.

 $H_{A}^{S}H_{A}^{S*}$  is strong enough,  $T_{BA}$  can be identified on the cross-correlation functions. In our definition, because both  $H_A^S(c)$  and  $H_B^S(c)$  are apparent velocity dependent, a FK filter or other velocity filter may be applied to suppress waves traveling at velocities lower or higher than desired apparent velocity. In the case the target reflector is nearly horizontal, for auto-correlation, because waves of interest travel vertically with nearly infinite C, a narrow band FK filter may be applied to suppress waves traveling at slow apparent velocities. For cross-correlation, the apparent velocities can be estimated for different common offset gather (COG), and an adaptive FK filter may be applied.

For cross-correlation, a velocity filter can be applied to suppress waves traveling at a lower or higher apparent velocities and thus to enhance the power spectrum of the sources  $(H_A^S H_A^{S*})$  or  $(H_B^S H_B^{S*})$  in equation (3). In our implementation, an adaptive FK (AFK) filter is applied to common offset gathers (COG). The passing band of the AFK is subject to the receiver offset and the estimated velocity model (defined by RMS velocity and the reflector depth, see Figure 2). Figure 4 shows the summation of cross-correlation functions from different processing. Without using AFK filter, it takes at least 500 hours to extract clear hyperbolic signals corresponding to the

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reflector. With the AFK filter estimated from the velocity model shown in Figure 2, 125 hours of data is sufficient (Figure 4b and 4c). Note that the FK filter also separates waves propagating from left to right (Figure 4b) and opposite direction (Figure 4c). The stronger one or the average of the two transfer functions corresponding to the two directions may be used for further analysis.

The AFK used here is dependent on a velocity model to estimate the pass-band for different COGs. Because zerooffset responses are cheaper to be extracted (25 hours for the example in Figure 3c) and they can be used to estimate reflector and homogenous media. In the real practice, a wide FK passband may be applied to account for small variations from these assumptions. Some priori information, such as the dipping of local geological structure, may also be taken into account to compute the proper FK filter passbands.

Other than the FK filter, other velocity filters, such as the linear  $\tau$ -p filter, may also be applied to enhance waves traveling along desired paths.



For the given source distributoin (1000 sources with SNR

Figure 3: Summation of auto-correlation functions for different time length: (a) 25 hours; (b) 250 hours; and (c) 25 hours of data with FK filter applied before auto-correlation. The passband of the FK filter is (-0.05 - 0.025 0.025 0.05)s/km. The horizontal bright spot is corresponding to the reflecor shown in the Figure 2. Every one hour of data conssts of 4 time segments.

velocity model for AFK passband computation. In fact, any model with a ratio of the reflector depth (H) and the RMS velocity (Vrms) equal to 1, i.e., H/Vrms=1, will result in a zero-offset reflection at 2 seconds as in Figure 3c. Figure 4e and 4f show the results with AFK filter estimated from a model with H=1.5km and Vrms=1.5km/s. Obviously, AFK filtering based on this model fails to bring up the hyperbolic signals as the true model does (Figure 4b and 4c). In practice, a group of possible models may be scanned to find the best results.

### **Discussions and conclusions**

An adaptive FK filter is introduced in this study to enhance seismic interferometry signal traveling with certain estimated apparent velocity. The examples shown in Figure 3 and 4 are based on the assumption of a horizontal and flat between 0.5 and 2.0 for every 15 minutes), 25 and 125 hours of data are needed to extract reflectors by autocorrelation and cross-correlation, respectively. But the time length needed to extract reflectivity from real field data may vary dependent on the real seismic activity in the study region. Nevertheless, with the AFK filter, time length needed to can be significantly reduced. We also find that the auto-correlation is a lot cheaper than the cross-correlation to extract the reflectivity, especially with the FK filter applied.

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